The Power of a Large-Area, High-\(z\) Galaxy Redshift Survey
to Reveal the Physics That Drove the Big Bang:

Inflation Model Constraints
and Other Powerful Ancillary Science Contributions

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I. Introduction

Understanding Inflation is fundamental to understanding the force(s) that shaped the observable Universe. During the interval between about $10^{-36}$ and $10^{-34}$ seconds after $t = 0$, Inflation is believed to have driven the Universe into faster-than-light expansion causing its size to grow by a factor of $\sim 10^{43}$ ($\sim 100$ e-foldings). In so doing, Inflation imparted the Universe with many of its most characteristic features, such as the:

- Initial density fluctuations which provided the seeds for galaxy formation and large-scale structure
- Isotropy of the cosmic microwave background (CMB) radiation
- Flatness of space

By providing a natural explanation for these otherwise unexplained features of our Universe, Inflation saves the Big Bang model. Yet, despite the indispensable role it plays in our understanding of the Universe, the physics that drove Inflation remains undetermined and largely unconstrained, although theoretical models abound. It is remarkable that current technology permits an experiment which probes in detail this brief, but defining, moment 13.7 Gyr ago. The information obtained from the large-area high-$z$ galaxy redshift survey discussed in this White Paper can significantly constrain Inflation models and, thus, the governing physics at this early time (and very high energies). In addition, as discussed below, this data set can also be used to either better constrain or detect the curvature of space, measure the effects of Dark Energy at high redshift, detect the mass of the neutrino, and determine the star formation history of the Universe, among many other ancillary science benefits.

II. Inflation and the Power Spectrum

Inflation is assumed to be driven by an as-yet-undiscovered form of energy with the property of negative pressure. The current observational constraints imply that the Inflation epoch occurred no earlier than $10^{-36}$ seconds after the birth of the Universe (or at an energy scale no higher than $10^{16}$ GeV). A series of phase transitions, occurring as the Universe cools, may govern the way in which Inflation unfolds. A scalar field, referred to as the inflaton field, and its associated potential, $V(\phi)$, control these phase transitions. The physics that drove Inflation dictates the shape of $V(\phi)$; measuring the shape of $V(\phi)$ makes it possible to determine the physics responsible for Inflation.

As Inflation ends, the energy density declines, locking in the shape of the power spectrum of primordial density fluctuations, $P_{\text{prim}}(k)$. Specifically, $P_{\text{prim}}(k)$ is related to the shape of $V(\phi)$ by the relations shown in Fig. 1, where the spectral index (or tilt) and curvature (or running) of the fluctuation power spectrum are directly related to the first, second, and third derivatives of the shape of the inflaton potential. If the expansion rate during Inflation were a constant, the resulting primordial power spectrum would be a pure power law with a scale-invariant spectrum, and the tilt, $n_s$, and running, $\alpha_s$ would be 1 and 0, respectively. However, since the expansion rate changes slowly during Inflation, we expect small but measurable differences from a pure power law in the primordial power spectrum.

Inflation models are testable because they make specific predictions for the expansion rate during Inflation. We can then derive detailed predictions for the tilt and running to compare to the measured values. Unfortunately, all viable models are nearly degenerate because they are indistinguishable within current measurement errors. The WMAP team has used precise measurements of the angular power spectrum of CMB fluctuations at large angular scales to constrain the shape of the power spectrum of the primordial fluctuations generated by Inflation. Fig. 2 shows the power spectrum of matter fluctuations,
Large-Area, High-z Galaxy Redshift Survey and the Physics of Inflation

**Fig. 1.** Left: An example of the shape of the inflaton potential, $V(\phi)$, associated with the inflaton field $\phi$. This potential is often referred to as chaotic Inflation potential. The shape of the inflaton potential is fixed by the physics driving Inflation. The shape of $P_{\text{prim}}(k)$ is calculable from the shape of $V(\phi)$. Right: $P_{\text{prim}}(k)$ is related to the shape of $V(\phi)$ by the relation shown, where $k$ is $1/(\text{length-scale})$ and $k_o$ is some pivot wavenumber. The parameters $n_s$ and $\alpha_s$ are referred to as tilt and running and the parameters $\varepsilon$, $\eta$, and $\xi$ determine the shape of $V(\phi)$.

which includes the WMAP best-fit cosmological parameters from year one of the mission, along with the contributions from lower-$z$ surveys, such as the Sloan Digital Sky Survey (SDSS), and other measurement techniques sensitive to larger $k$ (smaller spatial scales). Unfortunately, CMB missions cannot measure the power spectrum at small spatial scales, the low-$z$ galaxies sampled in the SDSS are too far from their primordial distribution to distinguish between most Inflation models, and Ly$\alpha$ studies are not sufficiently accurate.

Each spatial scale probes a different epoch in the Inflationary period, and the high-$z$ survey discussed here stands alone in accuracy at small spatial scales. Combined with the results obtained by WMAP and Planck at larger spatial scales, it’s possible to directly measure the expansion history during Inflation over the broadest range which provides an even more powerful test of Inflation models than a high-$z$ galaxy survey or CMB measurements alone. To illustrate the power of a large-area high-$z$ survey to study Inflation, we use as an example the expected results from the Cosmic Inflation Probe (CIP), whose scientific capabilities have been analyzed in detail previously. CIP is a space-based mission capable of conducting a 1,000 sq. degree galaxy redshift survey in H$\alpha$ between 1.8 and 5 $\mu$m, using a modest 1.5-meter aperture and limited in sensitivity by only the zodiacal background. CIP has been under study by NASA since 2004, most recently as an Astrophysics Strategic Mission Concept Study for a NASA Medium-Class mission. The CIP survey would detect > 100 million objects between a $z$ of 1.8 and 6.5 and would enable measurement of the galaxy power spectrum, $P(k)$, to better than 1% over length scales of 1 to 100 Mpc. The goal is to use these data to estimate the shape of the primordial power spectrum, convert it to a scalar potential, and compare this to the predictions made by various Inflation theories.

The power spectrum shown in Fig. 2 shows the $k$-range sampled by various techniques, including the range that would be covered by CIP, along with existing measurements. The fit used as an illustration in this figure is based on WMAP first-year data and other existing data at higher $k$. The fuzzy grey area on either side of the power spectrum fit encompasses the range of most Inflation model predictions and illustrates why it’s necessary to measure $P(k)$ to 1% in order to distinguish between nearly identical models, which is the goal of a high-$z$ galaxy redshift survey.
Fig. 2. Power spectrum (after Tegmark et al. 2004) modified to show the $k$-range sampled by various techniques, including the range that could be covered by CIP, along with existing measurement errors. The fit used as an illustration in the Tegmark figure is based on WMAP first-year data and other existing data at higher $k$. It is necessary to measure $P(k)$ to 1% in order to distinguish between models, which is the goal of the high-$z$ survey. (Note: Tegmark et al. use a convention wherein $k = \pi$/length-scale.)

Since successful Inflation models must have a nearly flat potential, they generally predict tilt $\sim 1$ and running $\sim 0$. Fig. 3 shows predicted values of tilt and running from a sample of Inflation models (e.g., Power-Law, Hybrid, Phase Transition) characterized by a single scalar field and which make the Universe flat (or nearly flat). The constraint set by WMAP (5-year) + SDSS BAO + supernovae data is shown, along with the constraints possible with Planck and with a 1,000 sq. degree $z = 1.8 – 6.5$ survey, such as would be performed by CIP. Combined with the CMB data, which sample different spatial scales and thus different epochs of Inflation, a large-area, high-$z$ survey will test if running $\neq 0$ to unprecedented accuracy ($\Delta \alpha_s = \pm 0.003 \, 2\sigma$; $\Delta n_s = \pm 0.005 \, 2\sigma$). If running $\neq 0$, the implication is profound and will challenge the simplest models of Inflation.

III. Observed-to-Primordial Power Spectrum

Retrieval of the primordial power spectrum (from the observed power spectrum) is necessary to the analysis described above. By determining the galaxy distribution at early times, i.e., high-$z$, the departures from the primordial distribution caused by gravitational evolution – the so-called non-linearities – are reduced. At the moment there is no consensus as to how to compute the galaxy power spectrum in the highly non-linear, i.e., $z \lesssim 2$, regime (even with numerical simulations); however, when the matter density fluctuations are only mildly non-linear (i.e., $\delta_m \lesssim 0.6$), 3rd-order cosmological perturbation theory, or even a higher-order scheme (renormalized perturbation theory), is expected to provide sufficiently accurate ($\lesssim 1\%$) corrections to non-linearities to enable the desired Inflation studies. This is especially true at relatively high redshifts, such as $z \gtrsim 2$. 
Fig. 3. The crosses mark the predictions of Inflation models that are characterized by a single scalar field and which make the Universe flat (or nearly flat). The large yellow ellipse represents the $2\sigma$ constraints placed on tilt and running set by combining WMAP (5-yr), SDSS BAO, and supernova data (after Komatsu et al. 2008). The crosses within the yellow ellipse are consistent with current data. The $2\sigma$ constraint ellipse possible with the CIP 1,000 sq. degree $z = 1.8–6.5$ survey, Planck (Baumann et al. 2008), and CIP plus Planck are also shown. (The placement of the Planck, CIP and CIP plus Planck constraint ellipses is arbitrary and intended only to show their size.)

Jeong & Komatsu (2006) have shown that 3rd-order perturbation theory provides accurate descriptions of the non-linear matter power spectrum, and Crocce & Scoccimarro (2008) have shown that renormalized perturbation theory is effective even deeper into the non-linear regime. Jeong & Komatsu (2009) have shown that 3rd-order perturbation theory also provides accurate descriptions of the non-linear galaxy power spectrum in real space in the weakly non-linear regime, including non-linear and stochastic galaxy bias. Most importantly, they have shown that perturbation theory can be used to extract the unbiased estimate of the distance scales from the galaxy power spectrum. Matsubara (2008) has derived the expression for the non-linear galaxy power spectrum in redshift space by including non-linear peculiar velocity effects using renormalized perturbation theory. This is an area of very active research and, at the current rate of progress, it’s expected that major non-linear effects will be understood and included in the theoretical calculations in the near future. Concurrently, numerical simulations will also improve, enabling us to push our understanding of the non-linear galaxy power spectrum deeper into the non-linear regime.

IV. Primordial Non-Gaussianity as a Probe of the Physics of Inflation

While the shape of the galaxy power spectrum provides tight constraints on the shape of the inflaton potential, $V(\phi)$, additional information may be needed to learn more precisely about the physics that governs inflation. In particular, the power spectrum captures the full information of the primordial fluctuations only when the fluctuations obey a Gaussian distribution. If fluctuations are non-Gaussian,
we need to go beyond the Gaussian correlations. The three-point function, or its Fourier transform, the bispectrum, vanishes for Gaussian fluctuations, and thus it provides a sensitive probe of a deviation from Gaussianity, i.e., non-Gaussianity.

It is often said that Inflation predicts that the primordial fluctuations are Gaussian; however, sensitive experiments can detect small amounts of non-Gaussianity produced in some Inflationary scenarios. In particular, in order for Inflation to produce only tiny amounts of non-Gaussianity below the detection limit, Inflation must satisfy all of the following conditions: (a) Inflation was driven by a single scalar field, and the fluctuations were generated by the same field, (b) the sound speed of the inflaton field was equal to the speed of light, (c) the scalar field was always slowly-rolling on a smooth potential, and (d) the scalar field was in the vacuum state when quantum fluctuations were generated. Violation of any of these conditions would result in detectable non-Gaussian signals (see, e.g., Bartolo, Komatsu, Matarrese & Riotto 2004 for a review).

The form of non-Gaussianity generated by multi-field Inflation models (i.e., violation of the condition (a)) can be understood simply by writing the primordial curvature perturbation in real space, $\Phi(x)$, as $\Phi(x) = \phi_g(x) + f_{NL}\phi^2(x)$, where $\phi_g$ is a Gaussian field. The latest limit from the WMAP 5-year data is $f_{NL} = 38 \pm 21$ (68% CL; Smith, Senatore & Zaldarriaga 2009). The Planck CMB data are expected to reach the 68% CL limit of $\Delta f_{NL} \simeq 4$, and the high-$z$ galaxy surveys can reach $\Delta f_{NL} \simeq 1$ (Carbone, Verde & Matarrese 2008). The other forms of non-Gaussianity can similarly be improved by a large-area high-$z$ galaxy redshift survey (Sefusatti & Komatsu 2007).

Convincing detection of primordial non-Gaussianity would be a breakthrough in our understanding of the physics of Inflation. High-$z$ galaxy surveys will be able to determine whether the physics of Inflation was very simple (all of the above conditions (a)–(d) are simultaneously satisfied), or was richer.

IV. Other Fundamental Science

Just as has been demonstrated with the Sloan survey at lower redshifts, data from a large-area high-$z$ redshift survey can be used for a multitude of studies of fundamental importance, such as:

**Space Curvature and Inflation:** Very small curvature is a robust prediction of Inflation. Since current constraints are derived from determinations of the angular-diameter distance to the CMB last-scattering surface, which is also affected by Dark Energy, they are limited by our understanding of Dark Energy. Measurements of luminosity or angular-diameter distances to redshifts in the matter-dominated era can greatly reduce this uncertainty. With a 0.4% measurement of the distance to $z = 3$ provided by CIP, combined with the CMB data, one can measure curvature to better than $10^{-3}$ (Knox 2006).

The standard prediction of Inflation is that the curvature of the observable Universe has to be approximately equal to the amplitude of quantum fluctuations, i.e., $\sim 10^{-5}$. However, recent developments in String Theory known as the “String Landscape” naturally give rise to space with slightly negative curvature (i.e., an open universe) that is just below the current limits of $\sim 10^{-2}$ (Freivogel et al. 2006). A $z = 1.8–6.5$ galaxy redshift survey would be able to measure or constrain the curvature to $4 \times 10^{-4}$. Detection of any curvature at the $10^{-3}$ level would rule out the standard models of Inflation and detection of a positive curvature (i.e., a closed universe) would rule out Inflation models arising from String Landscape. Conversely, detection of a slight negative curvature would confirm the String Landscape prediction and establish String Theory as a leading candidate to explain Inflation.
Neutrinos and Cosmic Structure: Non-relativistic neutrinos suppress the growth of structure on scales smaller than the velocity dispersion of neutrinos times the Hubble time. This scale-dependent suppression can confuse our determination of the shape of the primordial power spectrum in two ways – through its amplitude and its shape. The amplitude of suppression is sensitive to the total mass of all non-relativistic neutrino species, while the shape is sensitive to the number of non-relativistic neutrino species. It is the latter that is degenerate with the Inflation parameters.

Since the velocity dispersion of neutrinos redshifts and evolves as $\sigma_\nu \propto (1 + z)$, one can identify and remove the effect of velocity dispersion by measuring the galaxy power spectrum at different redshifts. The wider the redshift range observed, the better neutrino effects on the power spectrum can be removed. This process has been studied in detail by Takada et al. (2006), who have shown that a $z = 1 – 6.5$ survey can improve the determination of the number of non-relativistic neutrino species by as much as a factor of two, thus improving the constraints on the shape of the primordial power spectrum by as much as 50%. A high-$z$ survey will provide the best cosmological determination of the properties of neutrinos, including their mass and number. Current measurements from neutrino oscillation experiments place a lower bound on the neutrino mass of 0.05 eV. The CIP $2\sigma$ uncertainty on the mass will be 0.05 eV. CIP will therefore be able to measure the total neutrino mass (not just a lower limit) to better than $2\sigma$. In addition, the data contained within a high-$z$ galaxy redshift survey can be used to address the question of whether all of three – or just two – neutrino species are non-relativistic.

Dark Energy: A high-$z$ survey would contribute to the study of Dark Energy in several ways. First, because the measured expansion rate of the Universe depends on the space curvature, matter density, and Dark Energy ($\Omega_\Lambda$), by providing a more accurate constraint on the space curvature and matter density, the high-$z$ data will improve all measures of $\Omega_\Lambda$, including those focused on $1 \leq z \leq 2$. The CIP survey measures curvature and matter density to better than 0.1% and 0.5%, respectively, both of which are better than Planck can achieve. Second, if Dark Energy is not Einstein’s cosmological constant, i.e., $w \neq -1$, and varies with redshift, direct measures of $w(z)$ over as broad a range of redshifts as possible will be indispensable to understanding Dark Energy. A large-area high-$z$ survey would provide these data. Finally, the amplitude of the power spectrum provides a direct measure of the growth of structure, and thus helps to test whether Dark Energy is real, or a manifestation of an incomplete theory of gravity.

Star Formation History of the Universe: H$\alpha$ has proved to be a remarkably robust indicator of star formation rates, at high as well as low redshifts, even in the presence of extinction. With over $10^8$ H$\alpha$ measures from $1.8 < z < 6.5$, thus reaching into the end of the reionization epoch, a high-$z$ survey will provide the most accurate picture of star formation over the most critical epochs for galaxy formation. The implications for understanding galaxy formation and evolution are more fully explored in a separate White Paper submitted to the Galaxies Across Cosmic Time Panel.

V. Overlap with Other Approaches

The most powerful constraints on the shape of $V(\phi)$, and thus on Inflation, result from measuring the primordial power spectrum over the broadest range of $k$. Thus, a well-thought-out program to understand Inflation should include a set of experiments well suited to do this. COBE and WMAP have demonstrated the power of CMB measurements to constrain the shape of the power spectrum at large spatial scales, i.e., $k = 0.001$ to $\sim k = 0.1$ Mpc$^{-1}$. Planck and CMBPol will undoubtedly build upon these achievements. Specifically, Planck will add to WMAP’s impressive results through its greater
sensitivity and slightly higher spatial resolution. These instrumental improvements will allow Planck to measure tilt and running to the accuracy shown in Fig. 3. CMBPol will obtain limits on tilt and running comparable to those expected from Planck. As such, a high-z galaxy redshift survey, such as CIP’s, plus Planck or CMBPol reduces the uncertainties on tilt and running by as much as an additional factor of 3 to 4. The tighter constraints result from the increased $k$-range studied rather than any increased accuracy (which is why either Planck or CMBPol works as well in combination with CIP).

However, the primary focus of CMB polarization measurements will be an attempt to measure a different and important signature of Inflation, namely the B-mode polarization of the CMB induced by gravitational waves generated in the early Universe. Each Inflation model represented in Fig. 3 predicts a specific amount of the B-mode CMB polarization, which can be thought of as a third axis in this figure. Thus, B-mode polarization experiments provide an additional Inflation constraint. Unfortunately, among the many currently viable Inflation models there is a large range of predicted gravitational wave amplitudes; many models predict an amount of CMB polarization that falls below the sensitivity limit of CMBPol (e.g., Liddle & Lyth 2000). In addition to potentially undetectable levels of polarization, CMB measurements struggle with foreground contamination on small spatial scales, such as the Sunyaev-Zeldovich effect from clusters of galaxies, the Ostriker-Vishniac effect from reionization, and confusion due to low-z submillimeter and radio point sources. On large scales, where gravitational waves make the largest contribution, polarized dust and synchrotron emission from our Galaxy severely limits our ability to extract the primordial B-mode polarization. If Inflation occurred later than $10^{-34}$ seconds after the birth of the Universe (or energy scale lower than $10^{15}$ GeV), no B-mode polarization would be detected by CMBPol. Without doubt, the B-mode polarization signal would provide valuable new information about Inflation if it’s strong enough to be detected and if the degree of polarization can be reliably extracted from the complex foreground. Because the magnitude of these problems is presently unknown, the ability of CMBPol to advance our knowledge of Inflation beyond what will be contributed by Planck ranges from significant to relatively small. Because a large-area high-z galaxy redshift survey utilizes a well understood method, there is little doubt that it will strongly limit the range of viable Inflation models.

References: