Hubble Accelerations Described Relative to Mass Distributions in the Universe through the Use of Cosmological Kinematics

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Describing Real Kinematic Accelerations Relative to Mass Distribution in the Universe Rather than Relative to Absolute or Abstract Inertial Systems

Ordinarily, kinematic mechanics describes accelerated motion in terms of motion relative to inertial systems in which accelerations are due to "fictitious kinematic forces." However, on a cosmological scale, the universe is expanding so that the inertial systems at rest relative to the local Hubble drift within each galaxy are accelerating away from each other in accordance with Hubble's law. Therefore, no single universal inertial system exists. Any kinematics that would be observationally valid on a cosmological scale should then be defined without reference to inertial systems. Such cosmological kinematic equations would then describe real repulsive accelerated motion relative to the distribution of matter in the universe, rather than relative to an individual inertial system, and would be similar to the manner in which gravitation describes attractive accelerations without reference to inertial systems. The definition of kinematics in terms of motion relative to the mass distribution in the universe has been given little consideration within cosmology.

It is the purpose of this paper to propose that a very modest research program based on kinematics be created that describes the Hubble expansion in a manner that is similar to the accelerated expansion presently observed in the Type Ia supernovas. Although a very simple connection between the Hubble expansion and kinematic motion was pointed out by E. A. Milne[1] many years ago, it was not sufficiently mathematically developed so as to describe exactly how peculiar velocities evolve into Hubble motion. However, a detailed kinematic method was described by me in a paper[2] in 1993, but unfortunately published at a time when the acceleration of galaxies in the present universe and the existence of the cosmological constant were not generally thought to be possible. This white paper briefly shows how the Hubble parameter H can be generated in simplified models and describes how a balance between the outward kinematic acceleration and the inward gravitational acceleration could be achieved through a series of expansions and contractions, in which peculiar velocities cause each expansion to be larger than the one occurring before, such that they build one upon the other until a very near balance occurs.

Defining The Hubble Parameter H Relative to the Distribution of All Mass Particles in the Universe - Its Consequences in Cosmological Kinematics

Within general relativity the Hubble parameter is only defined in terms of the expansion parameter a in the form $H = \dot{a}/a$, where the dot over a letter indicates a derivative with respect to time t. A model, such as the Friedman-Lemaître model, describes the effects of gravity on $\dot{H}$ which changes H with respect to time, but does not define H itself because it only inserts an undefined cosmological constant when $\dot{H}$ is integrated. The Hubble parameter at any instant of time is actually a function of only the positions and velocities of mass particles in the universe. Gravity only directly affects $\dot{H}$ and defines such quantities as gravitational accelerations but does not directly define H.

A general description of the cosmological method of kinematics will now be explained here along with the manner in which it could be applied by cosmologists to calculate the exact manner in which the balance between gravitational attraction and the kinematic repulsion occurs.

A kinematic definition of the Hubble parameter H requires that it be expressed only in terms of the positions of mass particles and not photons. The reason for this is that the use of masses involves the rest mass of a particle which is independent of the inertial system in which it is located. However, the equivalent rest mass of a photon depends on the inertial system in which it's frequency and therefore energy is measured. Since the velocity of an inertial system at rest relative to the Hubble drift itself depends on the value of H, it is not then possible to use photons to define H.

It will be assumed that the universe may be considered to be divided into a finite number N of small but macroscopic mass particles of equal rest mass m. When N is large, any two unequal masses could be considered to be composed of different numbers of masses n_1 and n_2 to form the masses $n_1 m$ and $n_2 m$. The vector position $\mathbf{S}_k$ used here will always refer to the position any particle k and represents the metric three-dimensional length that can be physically measured from the center of the expansion to the position of particle k. The velocity of the particle is defined by

$$\dot{\mathbf{S}}_k = H\mathbf{S}_k + \dot{\mathbf{s}}_k, \quad (1)$$

where $\dot{\mathbf{s}}_k$ is the peculiar velocity of particle k. The unit vector along the direction of $\mathbf{S}_k$ is defined as $\mathbf{k} = \mathbf{S}_k/S_k$. The same letter k is used here to represent both the unit vector $\mathbf{k}$ as well as the subscript defining the particle's position, however they are easily distinguished because one is a vector and the other a scalar. By taking the scalar product of both sides of Eq. (1) with $\mathbf{k}$ and summing over all values of k from 1 to N an expression for the Hubble parameter is easily seen to be

$$H = (\Sigma\dot{\mathbf{S}}_k \cdot \mathbf{k})/(\Sigma\mathbf{S}_k \cdot \mathbf{k}) \quad (2a)$$

when the condition

$$\Sigma \dot{\mathbf{s}}_k \cdot \mathbf{k} = 0 \qquad (2b)$$

is satisfied. When $\mathbf{S}_k = \mathbf{r}_k a$ is substituted into Eq. (2a) it reduces to $H = \dot{a}/a$ when it is recognized that $\dot{\mathbf{r}}_k a = \dot{\mathbf{s}}_k$ which in turn satisfies Eq. (2b). If the mass of each particle were m and the sum in Eq. (2b) is multiplied by this mass it is seen that the equation represents a condition consistent with the conservation of momentum in Newton's third law of motion.

The physical cause of the change in H can be seen by finding an expression for $\dot{H}$. This is accomplished by taking the derivative of Eq. (1) with respect to time and using Eq. (1) again to eliminate $\dot{\mathbf{S}}_k$ to obtain

$$\ddot{\mathbf{S}}_\mathbf{k} = (\dot{H} + H^2)\mathbf{S}_k + H\dot{\mathbf{s}}_k + \ddot{\mathbf{s}}_k. \qquad (3)$$

Equation (3) is a general equation which by itself does not define either H or $\dot{H}$. Taking the scalar product of both sides of Eq. (3) with $\mathbf{k}$, while summing over all values, results in an equation which can be solved for H in the form

$$H = (\Sigma \ddot{\mathbf{S}}_k \cdot \mathbf{k})/(\Sigma \dot{\mathbf{S}}_k \cdot \mathbf{k}) \qquad (4)$$

when the condition

$$\dot{H} = (\Sigma \dot{\mathbf{s}}_k \cdot \dot{\mathbf{k}})/(\Sigma \mathbf{S}_k \cdot \mathbf{k}) \qquad (5)$$

is imposed, and when the time derivative of Eq. (2b) is seen to be

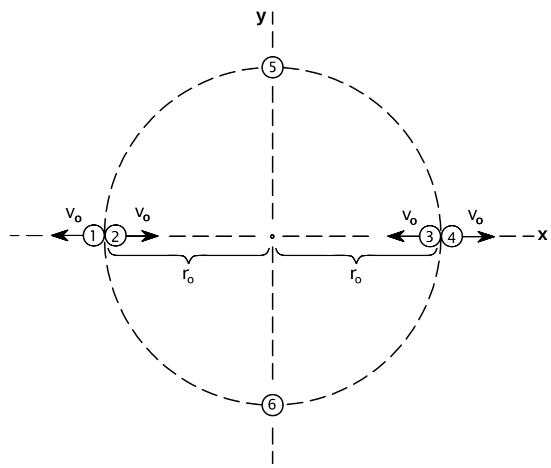
$$\Sigma \ddot{\mathbf{s}}_k \cdot \mathbf{k} = -\Sigma \dot{\mathbf{s}}_k \cdot \dot{\mathbf{k}}. \qquad (6)$$

Equation (5) shows that it is the peculiar velocities $\dot{\mathbf{s}}_k$ that generate the kinematic change in the Hubble parameter $\dot{H}$ in models where gravitational effects can be ignored. If there are no peculiar velocities, then H is constant.

The underlying physical principle of the kinematics used here is most easily seen as the peculiar velocities vanish so that H becomes constant and all the particles remain fixed in a configuration in which the relative distance between them remains constant while the angles subtended by any two particles relative to a third particle is also constant. In the manner of the Hubble motion then each particle considers itself to be "at rest" relative to all the other particles even though they are accelerating relative to each other. It is this configuration of constant angles by which nature defines the positions of inertial systems relative to the mass particles of the universe.

To illustrate how real kinematic accelerations physically generate Hubble motion, a simplified example of this will now be discussed in which only a few particles exist in a universe that has very small masses so that the force of gravitation is negligibly small and can be ignored. A model of the universe is considered here which at some initial time is arrayed in the configuration shown below in Fig. (a). All particles have equal mass and lie in the xy-plane at a distance r_0 from the center of the particle distribution. Initially the two pairs, particles 1 and 2, and particles 3 and 4, are located on the x-axis and have in the previous instant each just experienced equal and opposite mutual local forces that are of the same magnitude in the case of each pair in accordance with Newton's third law. They then have equal and opposite velocities $\dot{S}_k = v_0$. Particles 5, and 6 are located on the y-axis and are at this moment at rest relative to the center of the particle distribution. Before particles 2 and 3 pass through the origin, Eq. (2a) shows that $H = 0$ because as many particles are moving away from the center with velocity v_0 as are moving towards it so that $\Sigma\dot{\mathbf{S}}_k \cdot \mathbf{k} = 0$.

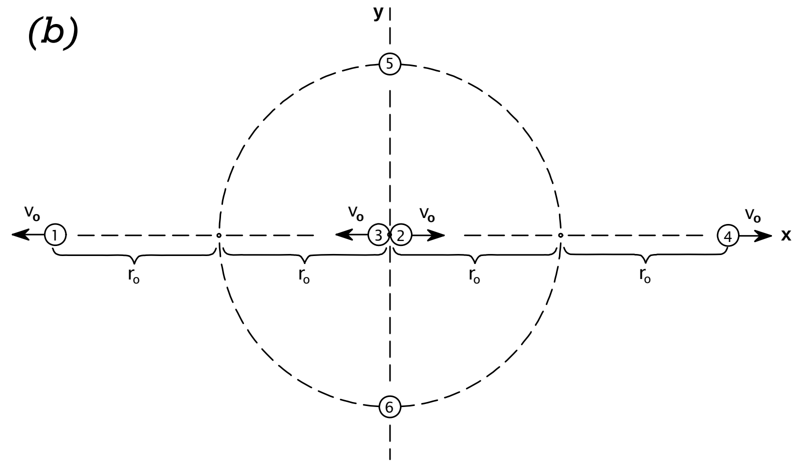
(a)



After a time each particle on the x-axis has moved with a constant velocity $\dot{S}_k = v_0$ through a distance r_0 relative to the origin of coordinates in the inertial system during the time when $H = 0$, and Fig. (b) shows that particles 2 and 3 have just passed by each other at the origin of coordinates so that Eq. (2a) then becomes

$$H = \frac{\Sigma\dot{\mathbf{S}}_k \cdot \mathbf{k}}{\Sigma\mathbf{S}_k \cdot \mathbf{k}} = \frac{4v_0}{6r_0} = \frac{2}{3}\frac{v_0}{r_0} \equiv \left(\frac{\Lambda}{3}\right)^{\frac{1}{2}}. \tag{7}$$

At the instant that the particles pass through the origin, the particles in motion are all moving away from each other and have moved into an expanding configuration so that the Hubble parameter changes from $H = 0$ to $H = 2v_0/(3r_0)$ which is a constant and acts as a cosmological constant $(\Lambda/3)^{1/2}$.



Note that if v_0 is much larger than r_0 then H would also be very large. After the expansion begins the behavior of the peculiar radial velocities v_k are described in the usual manner, as shown for example by Peebles,[3] as

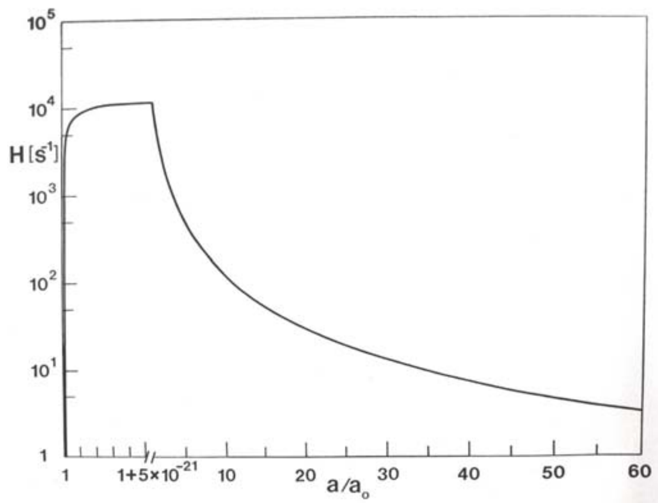
$$\dot{v}_k + Hv_k = 0, \tag{8}$$

whose solution is $v_k = v_{0k}\exp(-Ht)$, where v_{0k} is the initial peculiar velocity. Substituting this expression into Eq. (1) yields $\dot{S}_k = HS_k + v_{0k}\exp(-Ht)$. It is seen from this equation that in an expanding universe the peculiar velocities diminish with time when H is positive and the particles then approach the form of the Hubble recessional motion $\dot{S}_k = HS_k$. Each particle then experiences a real repulsive acceleration $\ddot{\mathbf{S}}_k = H^2\mathbf{S}_k$ when H is constant and when it has no peculiar velocity.

In the general case where gravitational acceleration exists in addition to the kinematic acceleration described above, Eq (5) can be shown to take the general form

$$\dot{H} = \frac{(\Sigma \dot{\mathbf{s}}_k \cdot \dot{\mathbf{k}})}{(\Sigma \mathbf{S}_k \cdot \mathbf{k})} - 4\pi G\left(\rho + \frac{p}{c^2}\right) + \frac{c^2}{R_0^2 a^2}. \tag{9}$$

The first term on the right-hand side of Eq. (9) is the kinematic term of Eq. (5) and the second and third terms are the gravitational terms contained in the Friedman-Lemaître cosmological equation. This is a general expression that defines the change in the Hubble parameter due to both kinematic and gravitational effects. A model illustrating how kinematic mechanics could generate an expansion of the universe having gravitation is shown in the graph below. This is a radiation model of the universe that initially consists of pure radiation and in which a very small number of particles are initially created within a single very small volume of space when $H = 0$. The total particle rest mass is negligibly small compared to the equivalent rest mass of the radiation. Mutual local forces between the particles create peculiar velocities equal to $c/10$ which in turn generate kinematic repulsions that cause an expansion of the universe with an inflationary form described by Eq. (9) and are shown in the graph below with H being a function of the expansion parameter a.



If a simplified model is considered in which at some single instant of time, occurring after the initial inflationary expansion, virtually all the radiation is converted into matter, then the expansion parameter can be shown to evolve into a form with the acceleration

$$\ddot{a} = \left(\frac{K}{2} \frac{a_0^4}{a^4} - \frac{4}{3}\pi G\rho + \frac{\Lambda}{3} \right) a. \tag{10}$$

The constants K and $\Lambda/3$ are both positive and can be shown to be defined in terms of physical quantities.

A Research Program for the Development of Mass Particle Kinematics in Cosmology

A realistic account of the expansion would require that the creation of mass particles occur over a wide expanse of time, rather than at one single instant as described in the model above, in order to describe how a balance occurs between the kinematic repulsion and the gravitational attractions as presently observed in the universe. The development of complex models of the universe that contain such a balance would require a group of cosmologists, in collaboration with elementary particle physicists, to use the repulsive kinematics described here to develop a balanced computer model. Such a balance is possible using the cosmological kinematic acceleration in Eq. (9) which is always positive due to the fact that $\dot{\mathbf{s}}_k \cdot \dot{\mathbf{k}}$ is always positive because the peculiar velocity $\dot{\mathbf{s}}_k$ always contains a component along the same direction of the change of its unit vector $\dot{\mathbf{k}}$. If the model consists of a series of expansions and contractions during which time peculiar velocities are created, a balance between gravity and the kinematic acceleration appears to be possible while peculiar velocities in Eq. (8) would diminish in the case of an expansion when H is positive, but would increase in a contracting universe in which H is negative. Such a universe would then not be expected to contract to as small a size as it had at the time of its origin in each previous expansion and would begin a new expansion from a larger size and with larger and more numerous initial peculiar velocities that, after a number of cycles, could reach a balance between the repulsive and attractive accelerations.

The difficulty in explaining the observed repulsive acceleration as well as the very close balance between the repulsive and attractive accelerations, shows that the present-day understanding of the fundamental physics in cosmology requires that new methods and concepts need to be introduced. The kinematic method described in this paper is based on the principle that it is the motion between particles in the universe, rather than motion relative to absolute local inertial systems, that has physical meaning on the cosmological scale and that such motion can produce the observed repulsive cosmological acceleration. A method such as this one that works flawlessly both physically and mathematically should be seriously considered as a possibility.

References

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