Mass Spectrum of the Galaxy

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ABSTRACT

The current Galactic census is rooted fundamentally in detection of luminous objects. As a result, the frequency of isolated dark objects, such as black holes, neutron stars, and old brown dwarfs is almost completely unconstrained. These dark objects trace star formation at a very early epoch. Microlensing is the only viable path toward a mass-based census. Unfortunately, routine microlensing detections yield only the Einstein timescale, which is a degenerate combination of the mass, distance, and speed of the lens. An interferometer in solar orbit could routinely yield two additional microlensing parameters, the Einstein radii projected on the sky and on the observer plane. Such an interferometer is therefore the only viable path to a mass-based Galactic census.

1. Goal: Unbiased Survey Based on Mass, Not Light

What would an unbiased census of Galactic objects, dark and luminous, reveal? At a minimum, it would yield the frequency of black holes (BHs), neutron stars (NSs), white dwarfs (WDs), and old brown dwarfs (BDs), which are either completely dark or so dim that they defy detection by normal methods. BHs and NSs are essentially the only discrete local tracers of high-mass star formation at early epochs, just as the much more luminous WDs already trace intermediate-mass star formation. At present the only evidence we have on BDs in old populations [from microlensing (Gaudi et al. 2008; Gould et al. 2009) and other sources (Burgasser et al. 2003)] suggests that they are unexpectedly common. Such a survey might also find a significant component of the dark matter, although the majority of dark matter cannot be in the form of compact objects (Alcock et al. 2000; Tisserand et al. 2007). The only known way to conduct such a census is to put a high-precision astrometry telescope in solar orbit.

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[1]Discolsure: This white paper arises from a collaboration of 12 people that has existed for 10 years

2. Microlensing: The Only Viable Approach

Masses of astronomical bodies can be measured only by the deflections they induce on other objects, typically planets and moons that orbit solar-system bodies, and binary companions that orbit other stars. Masses of luminous isolated field stars can be estimated from their photometric and spectroscopic properties by calibrating these against similar objects in bound systems. Hence, photometric surveys yield a reasonably good mass census of luminous objects in the Galaxy.

Dark objects like BHs are another matter. Mass measurements of isolated field BHs can be obtained only by their deflection of light from more distant luminous objects. Indeed, it is difficult to even detect isolated BHs by any other effect. However, to go from detection to mass measurement (and therefore positive identification) of a BH is quite challenging.

Gravitational microlensing experiments currently detect about 800 microlensing “events” per year. While this number will increase dramatically with next generation microlensing experiments and LSST, the number of bright ($V \lesssim 17$) events required for high-precision measurements, will not change significantly (Han 2008). Hence the feasibility of mass measurements does not depend on survey improvements. The vast majority of the “lenses” are ordinary stars, whose gravity deflects (and so magnifies) the light of a more distant “source star”. As the source gets closer to and farther from the projected position of the lens, its magnification A waxes and wanes according to the Einstein (1936) formula

$$A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}, \quad u(t) = \sqrt{u_0^2 + \left(\frac{t-t_0}{t_{\rm E}}\right)^2}, \tag{1}$$

where u is the source-lens angular separation (normalized to the so-called Einstein radius $\theta_{\rm E}$), t_0 is the time of maximum magnification (when the separation is u_0) and $t_{\rm E}$ is the Einstein radius crossing time, i.e., $t_{\rm E} = \theta_{\rm E}/\mu$, where μ is the lens-source relative proper motion. The mass M cannot be directly inferred from most events because the only measurable parameter that it enters is $t_{\rm E}$, and this is a degenerate combination of M, μ and the source-lens relative parallax $\pi_{\rm rel}$:

$$t_{\rm E} = \frac{\theta_{\rm E}}{\mu}, \quad \theta_{\rm E} = \sqrt{\kappa M \pi_{\rm rel}}, \qquad [\kappa \equiv 4G/(c^2{\rm AU}) \sim 8\,{\rm mas}\,M_\odot^{-1}]. \tag{2}$$

It follows immediately that to determine M, one must measure 3 parameters, of which only one ($t_{\rm E}$) is routinely derived from microlensing events. Another such parameter is $\theta_{\rm E}$, which could be routinely measured from the image positions, if it were possible to resolve their O(mas) separation. A third is the “microlens parallax” $\pi_{\rm E}$,

$$\pi_{\rm E} = \frac{\pi_{\rm rel}}{\theta_{\rm E}}. \tag{3}$$

Combining equations (2) and (3)

$$M = \frac{\theta_{\rm E}}{\kappa \pi_{\rm E}}, \tag{4}$$

implying that the mass can be extracted from $\theta_{\rm E}$ and $\pi_{\rm E}$ alone. See e.g., Gould (2000).

Just as $\theta_{\rm E}$ is the Einstein radius projected onto the plane of the sky, $\pi_{\rm E}$ is related to $\tilde{r}_{\rm E} \equiv {\rm AU}/\pi_{\rm E}$, the Einstein radius projected onto the observer plane. And just as $\theta_{\rm E}$ could in principle be measured by resolving the two images on the sky, $\pi_{\rm E}$ could be routinely measured by simultaneously observing the event from two locations separated by $O(\tilde{r}_{\rm E})$ (Refsdal 1966; Gould 1995). "Routine" measurement of both $\pi_{\rm E}$ and $\theta_{\rm E}$ is essential. As of today, there have been a few dozen measurements of these parameters separately (e.g., Poindexter 2005), but only one very exceptional microlensing event for which both were measured together with sufficient precision to obtain an accurate mass of a dark object (an old BD of mass $M = 0.05\,M_\odot$). This single detection hints that such objects may be substantially more common than currently believed (Gould et al. 2009).

3. Secure Inference Requires Interferometer in Solar Orbit

In fact, such routine measurement is possible by placing an accurate astrometric and photometric telescope in solar orbit. For current microlensing experiments carried out against the dense star fields of the Galactic bulge, $\pi_{\rm rel} \sim 20\,\mu$as, so for stellar masses, $\theta_{\rm E} \sim 300\,\mu$as and $\tilde{r}_{\rm E} \sim 15\,$AU. Hence, a satellite in solar orbit would be an appreciable fraction of an Einstein radius from the Earth, so the photometric event described by equation (1) would look substantially different than it would from the ground. From this difference, one could infer $\tilde{r}_{\rm E}$ (and so $\pi_{\rm E}$).

Determining $\theta_{\rm E}$ is more difficult. As mentioned above, this would be straightforward if one could resolve the separate images, but to carry this out routinely (i.e., for small as well as large values of $\theta_{\rm E}$) would require larger baselines than are likely to be available in next-generation instruments. Rather, one must appeal to a more subtle effect, the deflection of the *centroid* of the two lensed images. This deflection is given by (Miyamoto & Yoshii 1995; Hog et al. 1995; Walker 1995)

$$\Delta\boldsymbol{\theta} = \frac{\mathbf{u}}{u^2+2}\theta_{\rm E}, \tag{5}$$

where u is the source-lens separation in units of $\theta_{\rm E}$. Simple differentiation shows that this achieves a maximum at $u = \sqrt{2}$, for which $\Delta\theta = \theta_{\rm E}/\sqrt{8}$, roughly 1/3 of an Einstein radius. Hence, if the interferometer can achieve an accuracy of $O(10\,\mu{\rm as})$ *at the time when this deflection is the greatest*, then $\theta_{\rm E}$ can be measured to a few percent.

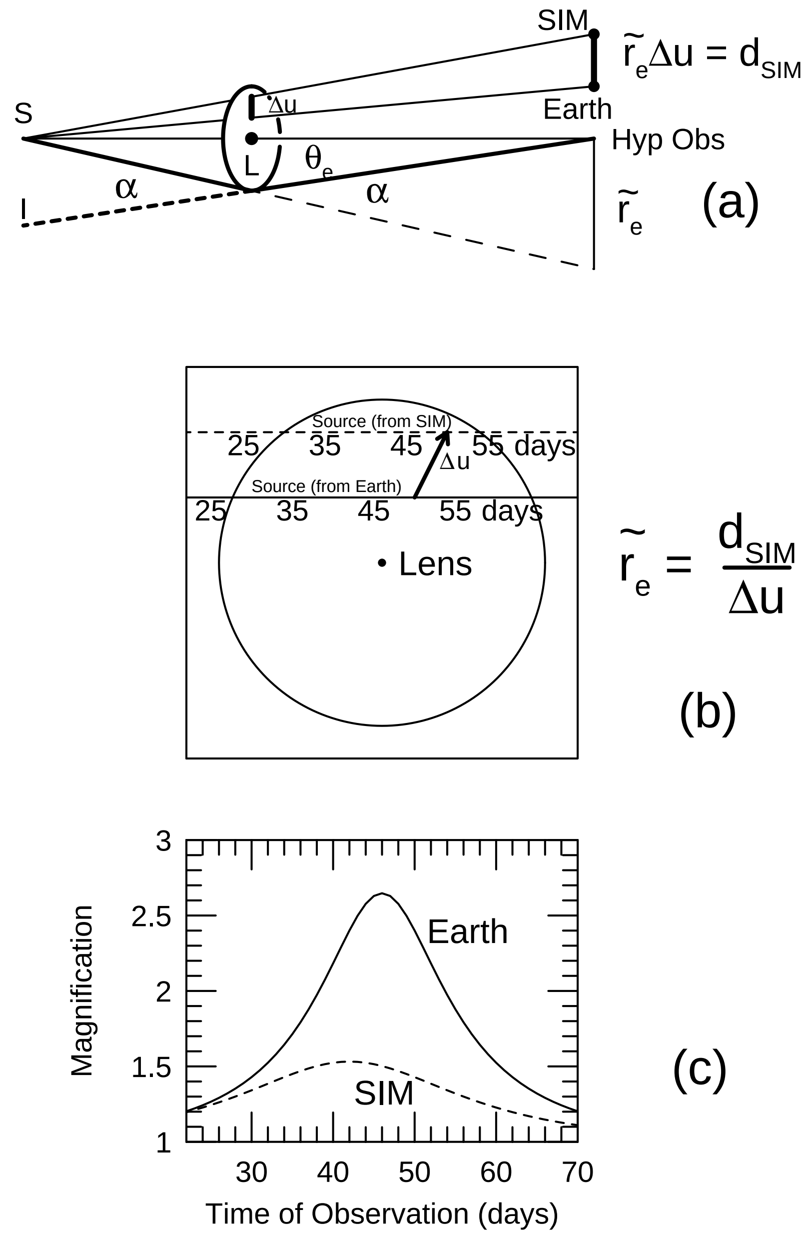

Fig. 1.— How an interferometer in Solar orbit measures masses. (a) Light from source (S) is deflected by lens (L) by angle α toward perfectly aligned hypothetical observer in a ring of Einstein radius θ_e, with phiyscal radius r_e (not labeled). $\tilde{r}_e$ is projection of r_e onto the plane of the observer. (b) The source position in the Einstein ring as a function of time (labeled in days) as seen from the Earth and satellite. The vector separation $\Delta\mathbf{u} = (\Delta t/t_e, \Delta u_0)$ remains a constant during the event, where t_e is the measured Einstein timescale and Δu_0 is the difference in impact parameters as seen from the Earth and satellite. (c) The resulting light curves as seen from the Earth and satellite allow one to measure the times of maximum t_0 and the peak magnifications (and so the impact parameters u_0), and hence Δu_0 and $\Delta t_0 = \Delta t$ and thus $\Delta\mathbf{u}$. From (a) it is clear that the magnitude of this vector, Δu, is the distance to the satellite $d_{\rm SIM}$ as a fraction of $\tilde{r}_e$. One therefore recovers $\tilde{r}_e = d_{\rm SIM}/\Delta u$. The centroid of light from the two images is deflected from the source position by $\boldsymbol{\theta}_e/(2+u^2)$, typically $\sim 100\,\mu$as. If the satellite has $\sim 10\,\mu$as precision, it can measure this deflection, and hence θ_e. The lens mass is then $M = c^2/(4G\tilde{r}_e\theta_e)$.

There are some subtleties as well as some challenges. Satellite measurements of $\tilde{r}_{\rm E}$ are subject to a four-fold discrete degeneracy, which can only be resolved by appealing to higher-order effects (Gould 1995). It is not enough to measure the centroid location to determine the astrometric deflection: one must also know the undeflected position to which the measured position is to be compared, and this can only be found by extrapolating back from late time astrometry. And the precision of the mass measurement depends directly on the signal-to-noise ratio of the underlying photometric and astrometric measurements. This is important because space-based astrometric telescopes are likely to be photon challenged and so to require relatively bright (and hence rare) microlensing events to provide accurate mass measurements.

Fortunately, the *Space Interferometry Mission (SIM)* can. Gould & Salim (1999) carried out detailed simulations based on the characteristics of *SIM* and concluded that with about 1200 hours of spacecraft time, it would be possible to make 5% mass measurements for about 200 microlenses. Most of these lenses will be stars, but at least few percent are likely to be BHs, and several times more are likely to be other dark or dim objects like NSs, old WDs, and old BDs. Since such a census is completely new, it may also turn up unexpected objects.

4. How Much of this Goal Can Be Achieved by GAIA?

Since *GAIA* will most likely be launched within a few years, one should ask whether it could carry out these measurements or whether it could somehow leverage its vastly larger number of targets to compensate for its inferior astrometric precision. And, of course, one should also ask how much of this program could be carried out from the ground.

Neither *GAIA* nor ground-based interferometers can address the integrated problem of measuring the entire mass spectrum of compact objects from BDs to BHs. However, GAIA could make some progress on the more limited (but very interesting) problem of measuring the frequency of BHs.

With regard to the full mass spectrum, *GAIA* has two problems: First, it will be in an L2 rather than a solar orbit, and it therefore cannot be used as one of two platforms (the other being the Earth) from which to measure the microlens parallax. Of course, this applies still more strongly to ground-based observations of any type.

Second, the astrometric precision required for reliable identification of "typical" lenses is substantially higher than will be achieved by *GAIA*. From equation (2), one finds that for $M = 1\,M_\odot$ and $\pi_{\rm rel} = 20\,\mu{\rm as}$ (typical of bulge lenses), $\theta_{\rm E} = 400\,\mu{\rm as}$. The maximum deflection of the centroid of light from the true source position is therefore $\Delta\theta_{\rm max} \sim 140\,\mu{\rm as}$, so that a $3\,\sigma$ detection requires a precision of $\sigma \sim 45\,\mu$as. Of course, *GAIA* is expected to achieve this precision for many stars, but this is *mission accuracy.* What is required here is to measure the $\theta_{\rm E}/\sqrt{8}$ "excursion" while it is actually happening, i.e., over roughly four Einstein crossing times, $4t_{\rm E}$ ($2\,t_{\rm E}$ while approaching and $2\,t_{\rm E}$ while receding). Hence, typically 120 days. For these short intervals, *GAIA* precision will be degraded by a factor $\sim \sqrt{\frac{5\,\rm years}{120\,\rm days}} \sim 4$. Since even the brightest sources (indeed the ones that *SIM* would observe) will be $V \sim 17$, the required precision is a factor of several beyond *GAIA*'s capability.

However, among the dark objects, BHs are probably the most interesting, and because BHs are substantially more massive than typical stellar lenses, BH events are longer and so are more susceptible to microlens parallax measurements from the ground (Bennett et al. 2002; Mao et al. 2002; Agol et al. 2002; Poindexter 2005). This obviates (or partially obviates, see below) the need for a satellite in solar orbit.

Moreover, since they are more massive than typical objects, BHs have larger Einstein rings, which dramatically improves the prospects for measuring them using *GAIA* At the same $\pi_{\rm rel} = 20\,\mu$as a "typical" BH of mass $M \sim 6\,M_\odot$ would have $\theta_{\rm E} \sim 1\,$mas and so $\Delta\theta_{\rm max} \sim 350\,\mu$as, so that a $3\,\sigma$ detection would only require a precision of $\sigma \sim 120\,\mu$as. And, the larger $\theta_{\rm E}$ implies a longer $t_{\rm E}$, typically 75 days, which reduces the degradation factor on *GAIA* astrometry from a factor 4 to a factor 2.5. That is, a "typical" BH would only require a "mission precision" of $\sigma \sim 120\,\mu{\rm as}/2.5 = 50\,\mu$as. If *GAIA* meets its design goals, this will be achievable for the brighter sources.

An important collateral point is that such *GAIA* measurements would break a common (and usually crippling) degeneracy in the ground-based microlens parallax measurements. Unlike trig parallax, microlens parallax is a vector, $\boldsymbol{\pi}_{\rm E}$, whose direction is that of the lens-source relative proper motion and whose amplitude is, of course, $\pi_{\rm E}$. For typical events, $\boldsymbol{\pi}_{\rm E}$ is not measurable at all (from a single observatory) because these events are too short. For very long events, it is completely measurable (e.g., Poindexter 2005), but for intermediate-length events, such as those expected for typical BHs, one component of $\boldsymbol{\pi}_{\rm E}$ (the one parallel to the projected position of the Sun-Earth axis) can be tightly constrained, while the orthogonal component is almost completely unconstrained (Gould et al. 1994). See, e.g., Jiang et al. (2003); Ghosh et al. (2003). By measuring the *direction* of the centroid displacement (which can be determined with roughly the same fractional precision as the amplitude of displacement) one can determine the *direction* of $\boldsymbol{\pi}_{\rm E}$ and so break the degeneracy.

Advances with ground-based interferometers are substantially more difficult than with *GAIA*, partly because they are restricted to brighter sources (which are extremely rare) and partly because of poorer precision.

In brief, neither *GAIA* nor ground-based interferometers have any hope of a complete census of compact objects (stars, BHs, NSs, WDs, and BDs). However, *GAIA* probably will make inroads toward a BH census (provided it reaches design specs), while ground-based interferometry may take a few initial steps toward this goal. Therefore, *SIM* provides the only viable approach on the current horizon to obtain an unbiased Galactic census.

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